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Book Review

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Spectral Methods in MATLAB

Llyod N. Trefethen, SIAM, Philadelphia, PA, 2000, 165 pp., $36.00, ISBN 0-89871-465-6

In many ways, this short book recently published by the Society for Industrial and Applied Mathematics (SIAM) in July 2000 could go unnoticed in the AIAA Community. In fact, for the few who might browse through the SIAM book titles, the presence of MATLAB in the title might be the only enticing factor in the book. And yet, spectral methods have been a powerful and popular computational tool for solving ordinary and partial differential equations for over 30 years, and these methods have been employed in solving a variety of problems in computational fluid dynamical problems in aerodynamics, turbulence, and transition. The literature on the subject is rich with fine and rigorous treatments of the subject. The question arises: What contribution a thin volume such as this can make to the field?

One observation is that the author is clear about the specific audience he would like to reach: the novice in the field of spectral methods who is still familiar with the basic concepts in numerical analysis and knows MATLAB. Therefore, the book's treatment of the subject is intentionally simplified. As mentioned in the preface, "the theoretical analysis is very limited and simple tools for simple geometries are emphasized rather than the "industrial strength" methods of spectral elements and *hp* finite elements." Also to give more focus to his treatment of the subject, the author focuses entirely on Chebyshev spectral collocation methods, and says nothing on Galerkin or Tau methods, which along with collocation, comprise the three basic spectral methods.

The second observation is on the use of MATLAB in illuminating the basic algorithms in the spectral collocation methods. In fact, the use of MATLAB and 40 short MATLAB programs is what sets this book and its treatment of spectral methods apart from its more detailed and theoretically rigorous predecessors. To explain the programs in the book, Trefethen warns the novice in a note that "the MATLAB programs in this book are terse," as one "can do an astonishing amount of serious computing in a few inches of computer code." This style of code writing seems to also dictate the whole tone of the book. "The best discipline for making sure you understand something is to simplify it, and simplify relentlessly." It is in this spirit that the book starts out to illuminate the basic concepts of spectral collocation by focusing on two principal ideas: interpolation of functions at selected nodes, and use of differentiation matrices for approximating the derivative of functions at these nodes.

The first chapter is devoted to differentiation matrices. Using simple examples, the author derives these matrices for finite difference techniques on equidistant gridpoints, and generalizes the idea to the periodic grids with trigonometric functions as interpolants (the basis of Fourier collocation schemes) and nonperiodic grid with uneven distribution with algebraic polynomials as interpolants (as in Chebyshev or Legendre collocation schemes).

The discussion on differentiation matrices leads to the description of Fourier spectral methods on unbounded grids in Chapter 2 and on periodic grids in Chapter 3. It is in Chapter 3 that an alternative way of calculating the derivatives via fast Fourier transform is explained. Chapters 5 and 6 continue the discussion by describing the Chebyshev differentiation matrices on Chebyshev grid points. Chapter 5 includes a set of results and theorems (stated without proof) that extend the results on accuracy of spectral methods from the Fourier methods (in Chapter 4) to Chebyshev collocation methods. This chapter also includes a very interesting explanation for the Runge phenomenon (divergence of interpolants on even grids near the endpoints of the interval) through the subject of potential theory. The effect of uneven distribution of Chebyshev points on accuracy of interpolation on these gridpoints is also explained.

Applications of the Chebyshev collocation methods appear for boundary value problems (homogeneous Dirichlet conditions in Chapter 7, nonhomogeneous and Neumann conditions in Chapter 11); calculation of eigenvalues and pseudospectra in Chapter 9; solution of partial differential equations in polar coordinates in Chapter 11 and 4th-order problems in Chapter 14. The examples and the MATLAB programs in Chapters 7 and 11 are particularly informative since they draw attention to collocation methods for boundary value problems.

The discussion on calculation of eigenvalues and pseudospectra leads naturally to the important issue of time stepping and stability region in Chapter 10. In the application of spectral methods to partial differential equations (PDE), the spatial part is discretized by spectral method, hence transforming the PDE to ordinary differential equations (ODE). To solve the resulting ODEs, some form of finite difference method is used. This chapter includes programs that demonstrate the stability region for the popular methods Adams-Bashforth, Adams Moulton, backward differentiation, and Runge–Kutta formulas.

Finally in Chapter 12, there is a discussion on the relationship between numerical quadrature and spectral methods. This discussion is of importance, especially in the context of solving optimal control problems where the cost functions that need to be discretized can be in integral form. Therefore, there is a need to unify the numerical scheme so that it can handle both the integral as well as the differential portion of the problem. The discussion here again centers on Chebyshev collocation scheme and the related Clenshaw-Curtis quadrature integration scheme. As the author points out, this scheme has inferior accuracy to using $N+1$ Gauss–Lobatto quadrature points; it yields exact interpolation for polynomials of degree $2N-1$, whereas the Clenshaw-Curtis method is exact for polynomials of degree N. This point is not of great concern to the author since he believes "most applications of spectral methods involve solution of differential equations." But for optimal control problems, this inadequacy of Chebyshev collocation methods can pose a problem. In fact, there is a solution to this problem that is not discussed in the text and that is to use Legendre points for collocation both in differentiation and integration. This observation leads to one important omission in the book (which is perhaps explained by the author's insistence on keeping the book as brief and focused as possible): that is, the use of Legendre polynomials. Other than the fact that the calculation of Legendre-Gauss–Lobatto points (as opposed to Chebyshev points) requires more complicated schemes, there is really no good reason for this omission. This short book successfully teaches the basic ideas of spectral methods through MATLAB programming in a clear, brief, and delightful style. This book may not be the main textbook on the subject; some concepts and examples may have to be supplemented by more detailed treatment from other texts, but with its clear presentation, good examples illustrated through MATLAB programs, and interesting exercises, it can certainly serve as the first book on the subject for beginners, who can then use the extensive bibliography to read and learn more about the rich field of spectral methods.

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